London Theory Institute
Lectures Series

Bogdan Stefanski
“ D-branes and boundary states ”

Informations
Pre-recorded Lectures : Youtube
Live Tutorial : Monday 7th of December, 11h30

Abstract

D-branes are a remarkable set of soliton-like objects in string theory, which have played a critical role in many areas of string theory. Despite being non-perturbative objects in string theory, they can be understood exactly using world-sheet methods through the boundary state formalism. In this lecture, I introduce the notion of a boundary state and show how to use it to extract physical information about D-branes and the open strings that live on them.

Exercices

Students are encouraged to try and solve the exercises by themselves and actively ask questions during the interactive live tutorial where solutions will be presented. The problems are :

1 Check that the momentum conjugate to X

$$P \equiv \frac{\partial S}{\partial \dot{X}} \, ,$$

satisfies the equal-time canonical commutation relation

$$\{X(\sigma_1)\, , \, P(\sigma_2)\} = i\delta(\sigma_1 - \sigma_2)\, ,$$

using the mode-expansions and commutation relations for α_n stated in the lecture.

2 Prove that for positive integers n, k

$$\alpha_n \, (\alpha_{-n})^k \, |0\rangle = n \, k \, (\alpha_{-n})^{k-1} \, |0\rangle \ .$$

3 Show that the Neumann boundary condition at $\tau = 0$ implies that the Fourier modes of X satisfy

$$p_0 = 0\,, \qquad\qquad \alpha_{-n} + \tilde{\alpha}_n = 0\,,$$

and that the corresponding boundary state is

$$|B\rangle = \mathrm{e}^{-\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}\tilde{\alpha}_{-n}}\,|B\rangle_{(0)}\;.$$

What is $|B\rangle_{(0)}$, the zero-mode part of the boundary state?

4 Expand the Born-Infeld action

$$S_{BI} = T_p \int d^{p+1}\mathrm{x}\;\mathrm{e}^{-\phi}\sqrt{-\det\left(g_{\mu\nu} + \mathcal{F}_{\mu\nu}\right)}$$

to quadratic order in fields for a D3-brane, using the expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\,,$$

where $\mu, \nu = 0, 1, 2, 3$. Take the antisymmetric $\mathcal{F}$ to have only non-zero components $\mathcal{F}_{01} = -\mathcal{F}_{10} = f_1$ and $\mathcal{F}_{23} = -\mathcal{F}_{23} = f_2$. Physically

$$\mathcal{F} \equiv B + F\,,$$

where B is the closed-string anti-symmetric NS-NS two form and F is the field-strength for the $U(1)$ gauge field living on a single D-brane.

5 Prove that for positive integers n, k

$$\alpha_{-n}\alpha_n\left(\alpha_{-n}\tilde{\alpha}_{-n}\right)^k|0\rangle = n\,k\,\left(\alpha_{-n}\tilde{\alpha}_{-n}\right)^k|0\rangle\;.$$

6 All fields that enter the string world-sheet action transform under the Virasoro symmetry. Write down, in terms of Fourier modes, how the (b, c) and $(\tilde{b}, \tilde{c})$ ghosts as well as the RNS fermions ψ^i and $\tilde{\psi}^i$ contribute to L_n and $\tilde{L}_n$. By using the symmetry preserving boundary conditions

$$\left(L_n - \tilde{L}_n\right)|B\rangle = 0\,,$$

deduce that the boundary conditions for the ghosts are

$$c_n = -\tilde{c}_{-n}\,, \qquad\qquad b_n = -\tilde{b}_{-n}\,,$$

and for fermions

$$\psi^\mu_r = \mp i\tilde{\psi}_{-r}\,, \qquad\qquad \psi^a_r = \pm\tilde{\psi}^a_{-r}\,,$$

for $\mu = 0, \ldots, p$ and $a = p+1, \ldots, 9$, with r half-integer/integer in the NS-NS/R-R sectors. Hint: To fix the relative sign between the μ and a components you will need to consider the symmetric boundary conditions for super-Virasoro generators. What does the extra $\pm$ freedom correspond to for the fermions ψ and $\tilde{\psi}$? Using these boundary conditions, write down the boundary state for ghosts and fermions. For this problem you may find this useful.

7 Show that the bosonic zero-mode contribution to the cylinder diagram for D-branes at positions y_1 and y_2 is given by

$$_{(0)}\langle B, y_1| \, e^{-2\pi l H_c} \, |B, y_2\rangle_{(0)} = l^{(9-p)/2} \mathrm{e}^{-(y_1-y_2)^2/(4l\pi)} \, .$$

Re-writing this in terms of the open string annulus modulus $t = \frac{1}{2l}$ explain the meaning of this term in terms of open strings ending on the D-branes.

8 Calculate the cylinder diagram between a Dp-brane and a Dq-brane and show that in the superstring case this vanishes for $p + q = 0$ mod 4.

9 Using a suitable orthogonal rotation, calculate the boundary state for a D-brane "at an angle" θ in the (X^8, X^9) plane. Calculate the cylinder diagram for D-branes at angles to one another along two planes, *i.e.* with θ_1, θ_2. For what values of θ_1 and θ_2 does this vanish?

References

Paolo Di Vecchia et al. "Classical p-branes from boundary state". In: *Nucl. Phys. B* 507 (1997), pp. 259–276. DOI: 10.1016/S0550-3213(97)00576-2. arXiv: hep-th/9707068

Joseph Polchinski and Yunhai Cai. "Consistency of Open Superstring Theories". In: *Nucl. Phys. B* 296 (1988), pp. 91–128. DOI: 10.1016/0550-3213(88)90382-3

Ahmed Abouelsaood et al. "Open Strings in Background Gauge Fields". In: *Nucl. Phys. B* 280 (1987), pp. 599–624. DOI: 10.1016/0550-3213(87)90164-7

Jr. Callan Curtis G. et al. "Adding Holes and Crosscaps to the Superstring". In: *Nucl. Phys. B* 293 (1987), p. 83. DOI: 10.1016/0550-3213(87)90065-4

Michael B. Green and Michael Gutperle. "Light cone supersymmetry and d-branes". In: *Nucl. Phys. B* 476 (1996), pp. 484–514. DOI: 10.1016/0550-3213(96)00352-5. arXiv: hep-th/9604091