London Theory Institute Lectures Series

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“ What is an anomaly ? ”

Informations
Pre-recorded Lectures :
Youtube
Live Tutorial :
Monday 25th of October,
10h30

Abstract

Four examples of an anomaly are presented, two from quantum mechanics and two from quantum field theory. The first example is a charged bead on a wire in the presence of a magnetic field. This example of a 't Hooft anomaly is related to the theta angle in Yang-Mills theory. The remaining three examples present scale and conformal anomalies. We will scatter a plane wave off an attractive delta function in two dimensions. We also look at a massless scalar field, both in two dimensions without a boundary and in three dimensions with one.

What is an Anomaly?

Lectures for New PhD Students
London Theory Institute
Problem Set

due Monday 25 October

1. **Charged Bead on a Wire with a Magnetic Field:** Consider a bead of mass m and charge q constrained to move on a circular wire of radius R. Concentric with the wire, there is a constant magnetic field of strength B in a disk shaped region of radius $R_0 < R$. Show that the quantum mechanical Hamiltonian for the bead takes the form

$$H = c\left(-i\frac{\partial}{\partial\theta} + \gamma\right)^2 .$$

   Express the constants c and γ in terms of m, q, B, R, and R_0.

2. **A Dihedral Group:** Show that the elements -1, R_π, and C, subject to the constraints $R_\pi^2 = \mathrm{id} = C^2$ and $CR_\pi C = -R_\pi$ generate the dihedral group D_8.

3. **Weyl transformations:** Verify the following Weyl transformation rules:

$$\begin{aligned}
\delta\Gamma^\lambda_{\mu\nu} &= \delta^\lambda_\mu\partial_\nu\omega + \delta^\lambda_\nu\partial_\mu\omega - g_{\mu\nu}\partial^\lambda\omega \ , \\
\delta n_\mu &= \omega n_\mu \ , \quad \delta n^\mu = -\omega n^\mu \ , \\
\delta K_{\mu\nu} &= \omega K_{\mu\nu} + h_{\mu\nu}n^\lambda\partial_\lambda\omega \ , \\
\delta K &= h^{\mu\nu}K_{\mu\nu} = -\omega K + 2n^\lambda\partial_\lambda\omega \ , \\
\delta R_{(2d)} &= -2\omega R_{(2d)} - 4\Box\omega \ , \quad \delta\Box_{(2d)} = -2\omega\Box_{(2d)} \ .
\end{aligned}$$

   A Weyl transformation acts on the metric as $g_{\mu\nu} \to e^{2\omega(x)}g_{\mu\nu}$. Here n_μ is a unit normal to the boundary, $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ is a projector onto the boundary, and $K_{\mu\nu} = h_{\mu\lambda}\nabla^\lambda n_\nu$ is the extrinsic curvature.

4. **The Conformally Coupled Scalar:** Find the value of the constant ξ that makes the following scalar field action Weyl invariant:

$$S = \int d^dx\,\sqrt{\det g}\,\left[(\partial_\mu\phi)(\partial^\mu\phi) + \xi R\phi^2\right]$$

   where R is the Ricci scalar curvature. Assume the Weyl transformation rules $g_{\mu\nu} \to e^{2\omega(x)}g_{\mu\nu}$ and $\phi(x) \to e^{-\frac{d-2}{2}\omega(x)}\phi(x)$.

The following two questions assume a normalization of the boundary anomaly coefficients that follows from the following anomalous scale variation of the effective action

$$\delta_\sigma\mathcal{W} = \int d^2x\,\sqrt{\det h}\,\left(aR_{(2d)} + b\hat{K}_{\mu\nu}\hat{K}^{\mu\nu}\right)\sigma$$

where $R_{(2d)}$ is the Ricci scalar curvature on the boundary and $\hat{K}_{\mu\nu} = K_{\mu\nu} - \frac{1}{2}h_{\mu\nu}K$ is the traceless part of the extrinsic curvature.

5. ***Boundary "a" for the Free Scalar:** Compute the boundary coefficient a for a conformally coupled three dimensional scalar field by computing the partition function on a hemisphere with both Dirichlet and Neumann boundary conditions.

6. ***Boundary "b" for the Free Scalar:** Compute the boundary coefficient b for the conformally coupled three dimensional scalar by computing the two point function of the displacement operator, both in the case of Neumann and Dirichlet boundary conditions.