London Theory Institute Lectures Series

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“ Witten diagrams and the Mellin transform in AdS/CFT ”

Informations
Pre-recorded Lectures : Youtube
Live Tutorial : Monday 23th of November, 10h30

Abstract

AdS/CFT duality (or more general holographic principle) represents a major advance in understanding quantum gravity, and provides powerful tools for studying strongly coupled quantum field theories. This talk will give a basic introduction to AdS/CFT duality, with the focus on the computation of Witten diagrams and their Mellin transform. Witten diagrams, which play the role of Feynman diagrams, provide the means for computing correlation functions in AdS/CFT. We will show that CFT correlation functions obtained from Witten diagrams have much simpler structures after Mellin transform. Correlators in Mellin space are very analogous to the flat-space scattering amplitudes, and they are often called Mellin amplitudes. We will demonstrate the ideas by studying a few non-trivial examples.

Exercices

Students are encouraged to try and solve the exercises by themselves and actively ask questions during the interactive live tutorial where solutions will be presented. The problems are :

1. Show the following integral identities. They are relevant in the computation of Witten diagrams.
   - Prove the following integration over a AdS bulk point
   $$\int_{\text{AdS}} dX e^{2Q\cdot X} = \pi^h \int_0^\infty \frac{dz}{z} z^{-h} e^{-z+Q^2/z}\,, \tag{1}$$
   where X is a bulk point in AdS_{d+1}, which can be parametrised as $X = \{\frac{1+x^2+z^2}{2z}, \frac{x^\mu}{z}, \frac{1-x^2-z^2}{2z}\}$.

- Another integral identity we use regularly for Mellin amplitudes is the Symanzik "star formula",

$$\int_0^\infty \prod_{i=1}^{n} \frac{dt_i}{t_i} t_i^{\Delta_i} e^{2\sum_{i<j} t_i t_j P_i \cdot P_j} = \frac{1}{2} \int_{-i\infty+c}^{i\infty+c} \prod_{i<j} \frac{d\gamma_{ij}}{2\pi i} \Gamma(\gamma_{ij}) (-2P_i \cdot P_j)^{-\gamma_{ij}} , \tag{2}$$

  with γ_{ij} satisfying the constraints $\sum_{j \neq i} \gamma_{ij} = \Delta_i$.

2. Compute the Mellin amplitude of a n-point contact Witten diagram in AdS_{d+1}

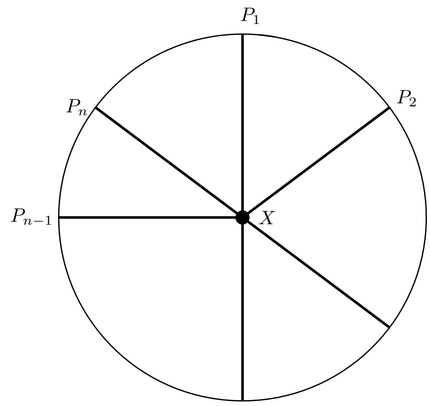

Figure 1: Here the contact interaction is $\lambda\, \phi_1 \phi_2 \cdots \phi_n$.

3. Consider a similar n-point contact Witten diagram but now derivatives:

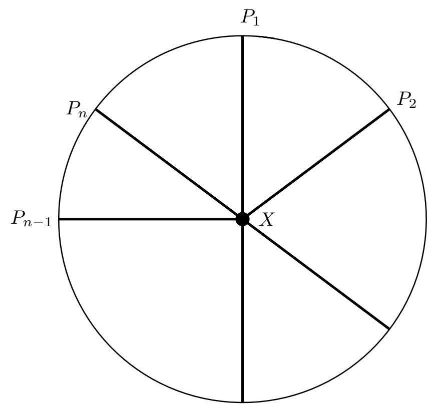

Figure 2: Here the contact interaction is $\lambda\, \nabla\phi_1 \cdot \nabla\phi_2 \phi_3 \cdots \phi_n$.

Compute the Mellin amplitude of the above contact diagram in AdS_{d+1} using the covariant derivative in the embedded space,

$$\nabla_A \phi = (\eta_A^B + X_A X^B) \frac{\partial}{\partial X^B} \phi \,. \tag{3}$$

4. Compute the Mellin amplitude of an exchange 4-point Witten diagram in AdS_{d+1} :

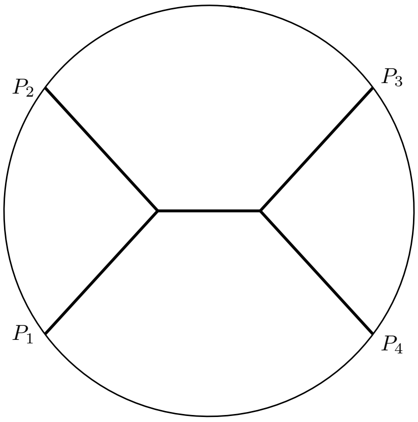

Figure 3: Here the three-point vertex is $\lambda\, \phi_1 \phi_2 \phi_3$.

Does the result have the structures of scattering amplitudes ?

References

Joao Penedones. "Writing CFT correlation functions as AdS scattering amplitudes". In: *Journal of High Energy Physics* (2010). eprint: arXiv:1011.1485

Joao Penedones. "TASI lectures on AdS/CFT". in: *New Frontiers in Fields and Strings* (2016). eprint: arXiv:1608.04948