London Theory Institute Lectures Series

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“ (Holographic) Fermi surfaces ”

Informations
Pre-recorded Lectures :
Youtube
Live Tutorial :
Monday 1st of November, 10h30

Abstract

In this lecture, we present a few elementary facts about Fermi surfaces, then discuss how to find interesting "non-Fermi liquids" via the AdS/CFT correspondence. We study different backgrounds (e.g. AdS, BTZ, and Reissner-Nordstrom), and the wave-equation of probe fields on top of these geometries. We discuss how to compute boundary two-point functions by solving the bulk equations and then explore the results.

(Holographic) Fermi surfaces

Exercises

1) What is $1\,\mathrm{eV}$ in kelvins? (This is considered to be a UV scale in condensed matter physics.)

2) AdS_d can be defined by the (universal cover of the) hyperboloid

$$X_{-1}^2 + X_0^2 - \sum_{i=1}^{d} X_i^2 = L^2$$

where L is the radius of AdS. The equation is solved by

$$X_{-1} = \frac{Lt}{z}\,, \qquad X_0 = \frac{z}{2} + \frac{L^2 + \vec{x}^2 - t^2}{2z}$$

$$X_i = \frac{Lx^i}{z} \qquad \text{for } i = 1, \ldots, d-1$$

$$X_d = \frac{z}{2} - \frac{L^2 - \vec{x}^2 + t^2}{2z}$$

$(t, z, \vec{x})$ are the Poincaré patch coordinates.

Show that the induced metric is given by

$$ds^2 = L^2\,\frac{dz^2 - dt^2 + d\vec{x}^2}{z^2}$$

3) Show that the metric

$$ds^2 = L^2\,\frac{dz^2 - dt^2 + d\vec{x}^2}{z^2}$$

satisfies Einstein's equations with a negative cosmological constant Λ. How is Λ related to L? For the calculations, I recommend using Mathematica with Matt Headrick's differential geometry package that you can find at:

`https://people.brandeis.edu/~headrick/Mathematica/index.html`

4) Show that the planar AdS-black hole metric

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2}\frac{dr^2}{f(r)} \tag{1}$$

with $f(r) = 1 - \left(\frac{r_0}{r}\right)^d$ satisfies Einstein's equations with a negative cosmological constant. [Note that the formula in the video has a typo: it should be f in the denominator, not f^2.]

5) Compute the temperature of (1) by rotating it into Euclidean signature and demanding that the geometry does not have a conical singularity at $r = r_0$.

6) The wave equation for a scalar field $\varphi(T, X, \rho) = e^{-ik_T T + ik_X X}\varphi(\rho)$ in the BTZ geometry turns out to be

$$\varphi''(\rho) + 2\coth 2\rho\, \varphi'(\rho) + \left(\frac{k_T^2}{\sinh^2 \rho} - \frac{k_X^2}{\cosh^2 \rho} - m^2\right)\varphi(\rho) = 0.$$

Solve the wave equation, find the ingoing solution at the horizon, and compute the retarded 2-point function by expanding the ingoing solution in the UV. Hint: a coordinate change $z = \tanh^2 \rho$ might help solving the equation.

7) The result should be

$$G_R(k_T, k_X) \propto \frac{\Gamma\left(\frac{\Delta_+}{2} - i\frac{k_T + k_X}{2}\right)\Gamma\left(\frac{\Delta_+}{2} - i\frac{k_T - k_X}{2}\right)}{\Gamma\left(\frac{\Delta_-}{2} - i\frac{k_T + k_X}{2}\right)\Gamma\left(\frac{\Delta_-}{2} - i\frac{k_T - k_X}{2}\right)},$$

where $\Delta_\pm = 1 \pm \sqrt{1 + m^2}$. Plot the function on the complex k_T plane at a fixed value for k_X. Are all the poles on the lower-half plane as required for a retarded Green's function?

8) Show that at $T = 0$ the near-horizon geometry of the planar AdS-Reissner-Nordström black hole is $AdS_2 \times \mathbb{R}^{d-1}$. [Note that the formula in the video has a typo: it should be f in the denominator, not f^2.]

9) Using Mathematica, numerically compute the boundary retarded Green's function of an operator dual to a charged scalar field in the AdS-Reissner-Nordström geometry. (You can set $d = 3$ so that the geometry is asymptotically AdS$_4$.) Can you find poles near $\omega = 0$ and $k \sim \mathcal{O}(1)$? Since bosons do not form a Fermi surface, what do these findings mean?

10) Compute the retarded Green's function of a charged scalar field in the (0+1)-dimensional "CFT dual" to the AdS$_2$ geometry

$$ds^2 = \frac{L_2^2}{\zeta^2}(-d\tau^2 + d\zeta^2) \qquad A_\tau = \frac{a}{\zeta} \tag{2}$$

where (τ, ζ) are the AdS$_2$ coordinates, A_τ is the only non-zero component of the gauge field, a is a constant, and L_2 is the AdS$_2$ radius.

For thermal QFT you can consult the (old) book by Fetter-Walecka: "Quantum Theory of Many-Particle Systems" or the book by Mahan: "Many-particle physics".

For the Lorentzian AdS/CFT prescription see [1]. A comprehensive "AdS/CMT" review is in [2]. The BTZ black hole is described in [3, 4]. For holographic Fermi surfaces, see [5, 6, 7].

References

[1] D. T. Son and A. O. Starinets, *Minkowski-space correlators in AdS/CFT correspondence: Recipe and applications*, *JHEP* **09** (2002) 042 [`hep-th/0205051`].

[2] S. A. Hartnoll, A. Lucas and S. Sachdev, *Holographic quantum matter*, `1612.07324`.

[3] M. Banados, C. Teitelboim and J. Zanelli, *The Black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849–1851 [`hep-th/9204099`].

[4] M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, *Geometry of the (2+1) black hole*, *Phys. Rev.* **D48** (1993) 1506–1525 [`gr-qc/9302012`]. [Erratum: Phys. Rev.D88,069902(2013)].

[5] H. Liu, J. McGreevy and D. Vegh, *Non-Fermi liquids from holography*, `0903.2477`.

[6] T. Faulkner, H. Liu, J. McGreevy and D. Vegh, *Emergent quantum criticality, Fermi surfaces, and AdS2*, `0907.2694`.

[7] M. Cubrovic, J. Zaanen and K. Schalm, *String Theory, Quantum Phase Transitions and the Emergent Fermi-Liquid*, *Science* **325** (2009) 439–444 [`0904.1993`].